Sheet 4

Please email your solution to your corresponding instructor before **Friday, March 20th** at **23h59** by an email with object “<First name> <Last name> Stochastik Sheet 4” and an attachment “<First name>_<Last name>.pdf”.

Exercise 1. One could think that the day of the week, or the date, of birth of an individual picked randomly (uniformly) in the population follows a uniform distribution. Find factors explaining why it is not the case (and cite your sources!).

Exercise 2. A new, deadly virus is spreading among the population. Its propagation is as follows: at an instant $t \in \mathbb{N}_0$, there is a certain number $N(t)$ of viruses alive. Between time t and $t+1$, each virus dies and creates new viruses following a probabilistic law: there is a probability measure $\mathbb{P}$ on $\mathbb{N}_0$ such that, in an independent manner, each virus has a probability $\mathbb{P}(\{k\})$ of making k new viruses.

1. Find how the expectation of the number of viruses evolves with time. This evolution will involve the quantity $E = \sum_{k=0}^{+\infty} \mathbb{P}(\{k\})$.

2. Thanks to containment measures and thorough washing of hands, $E < 1$. Prove that, almost surely, the epidemic disappears in finite time.

(Remark: if $E > 1$ one can prove that, with non-zero probability, the virus never goes extinct, but this is much harder!)

Exercise 3. A pharmacological lab has developed a new product for testing people against this virus. The rate of false positives (if you are not infected, what is the chance that the test says you are infected) is 2% and the rate of false negatives (if you are infected, what is the chance that the test says you are not infected) is 5%.

There are 8 million inhabitants in Switzerland and 200 of them are infected with the virus (and don’t know about it). Everyone goes to their doctor to be tested. If you are tested positive, what is the chance that you are infected?

Exercise 4. A student works from home because the courses have been cancelled. To be well-prepared for the exam, they solve every day a random number of exercises, following a fixed probability distribution on $\mathbb{N}_0$. We know that the expectation of the number of exercises they solve every day is 50.

1. Give an upper bound for the probability that, on a given day, they solve more than 75 exercises.

2. We now know that the standard deviation (square root of variance) of the number of exercises solved by day is 5 exercises. Improve the previous upper bound.

3. Can one give a lower bound for the probability to solve more than 75 exercises?

Exercise 5. For the following examples, write down carefully the probability space, its measure, and the random variable(s) of interest. Then, solve the problem as much as possible.

1. Alice flips two coins and bets on the fact that the two coins will fall on different sides.

2. Bob tosses a fair dice, then flips as many coins as the number indicated by the dice; he counts the number of coins that fall on heads.

3. Caroline tosses a coin until it falls on heads, and counts the number of tries.

4. Davide takes k loto balls out of an urn containing n balls numbered from 1 to n. He counts the sum of the numbers indicated by the balls.

5. Eskaterina places n tickets numbered from 1 to n into n envelopes numbered from 1 to n (one ticket per envelope). She counts the number of tickets that share the same number as the envelope they are placed in.